Método das secantes

ordem de convergência $\beta = \frac{1+\sqrt{5}}{2} = 1.618\ldots$

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

$$e_{n+1} = x_{n+1} - a$$
$$e_n = x_n - a$$

$$e_{n+1} = e_n - \frac{f(x_n)(e_n - e_{n-1})}{f(x_n) - f(x_{n-1})}$$

$$e_{n+1} = \frac{\cancel{e_n f(x_n)} - e_n f(x_{n-1}) \cancel{- e_n f(x_n)} + f(x_n)\, e_{n-1}}{f(x_n) - f(x_{n-1})}$$

$$e_{n+1} = \frac{e_{n-1} f(x_n) - e_n f(x_n)}{f(x_n) - f(x_{n-1})} =$$

$$= \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \left[\frac{1}{x_n - x_{n-1}}\right] \left(e_{n-1} f(x_n) - e_n f(x_{n-1})\right)$$

$$= \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \frac{e_{n-1}\, e_n}{x_n - x_{n-1}} \left(\frac{f(x_n)}{e_n} - \frac{f(x_{n-1})}{e_{n-1}}\right)$$

$$= \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} e_{n-1} e_n \left\{ \frac{\frac{\cancelto{0}{f(a)} + e_n f'(a) + e_n^2 f''(a)/2 + \cdots}{e_n} - \frac{\cancelto{0}{f(a)} + e_{n-1} f'(a) + \cdots}{e_{n-1}}}{x_n - x_{n-1}} \right\}$$

$$e_{n+1} = \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}\, e_{n-1} e_n \, \frac{\cancel{f'(a)} + e_n f''(a)/2 + \dots - \cancel{f'(a)} + \frac{f''(a)}{2} e_{n-1}}{x_n - x_{n-1}}$$

$$= \underbrace{\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}}_{\approx f'(a)}\, e_{n-1} e_n \, \cancel{\frac{(e_n - e_{n-1})}{x_n - x_{n-1}}}\, \frac{f''(a)}{2}$$

$$e_{n+1} = \underbrace{\frac{f''(a)}{2 f'(a)}}_{A}\, e_{n-1} e_n$$

Buscamos uma expressão na forma:

$$\left.\begin{aligned} e_n &= B\, e_{n-1}^{\beta} \\ e_{n+1} &= B\, e_n^{\beta} \end{aligned}\right\} \quad e_{n+1} = B\left[B\, e_{n-1}^{\beta}\right]^{\beta}$$

$$\cancel{B}\left[B\, e_{n-1}^{\beta}\right]^{\beta} = A \cdot \cancel{B}\, e_{n-1}^{\beta} \cdot e_{n-1}$$

$$e_{n-1}^{\beta^2 - \beta - 1} = \frac{A}{B^{\beta}} = cte$$

$$A^{\frac{1}{\beta}} = B \implies \beta^2 - \beta - 1 = 0 \implies \beta = \frac{1 + \sqrt{5}}{2}$$